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Husserl, Jacob Klein, and Symbolic Nature

Joseph Cosgrove

Husserl's philosophy of science in *The Crisis of European Sciences* focuses on the reactivation of the sedimented meanings by which science, as a positive historical phenomenon, is constituted.[1] Although the historical emphasis of *The Crisis* is arguably implicit in the "genetic phenomenology" of earlier works such as *Formal and Transcendental Logic*, the "ideal sense-histories" sought after via the earlier genetic method must now be grounded in actual historical research. For, in the absence of an understanding of its historical origins, maintains Husserl, "science as given in its present-day form ... is mute as a *development of meaning.*"[2] Nevertheless, this reconceived historical phenomenology remains distinct from history in the usual sense, or even philosophy of history, in that it still situates itself within the phe-nomenological reduction, where the aim is the reactivation of an *ideal sense-history* as opposed to historical knowledge per se.

For Husserl, modern, or "Galilean," science is characterized by a mathematical idealization of nature, the "surreptitious substitution of idealized nature for prescientifically intuited nature."[3] The phenomeno-logical desedimentation of this science requires the excavation of two specific strata in its genetic constitution: the geometrical idealization of nature which Husserl associates most closely with Galileo himself; and, deposited over the latter, as it were, the *symbolic* idealization of nature via algebraic formalization.

In this essay, I shall direct my attention to the second layer of sedimentation identified by Husserl, that of symbolic idealization via algebraic formalization.[4] This would indeed seem to be in line with Husserl's basic intention, which has to do not so much with the fact that so-called "Galilean science" uses mathematics as with *how* it uses mathematics; namely, as *mathesis universalis* in the form of a "self-enclosed, coherent systematic theory . . . proceeding from axiomatic concepts and propositions," which anticipates a universal science, the "one all-encompassing science, the science of the totality of what is."[5]

Husserl describes this substitution of mathematical idealities for the real world in terms of a kind of "reification of method":

> Mathematics and mathematical science, as a garb of ideas, or the garb of symbols of the symbolic mathematical theories, encompasses everything which, for scientists and the educated generally, *represents* the life-world, *dresses it up* as "objectively actual and true" nature. It is through the garb of ideas that we take for *true being* what is actually a *method.*[6]

Discovering the way modern mathematical science has come to take its method of representation for the true being of nature is thus the essential task of the historical phenomenology projected and roughly sketched in *The Crisis.*

As Burt Hopkins points out, Jacob Klein carried out, in significant measure, the historical research called for by Husserl's *Crisis* but not carried out by Husserl himself.[7] Klein, in his *Greek Mathematical Thought and the Origin of Algebra,* uncovers a transformation in the intelligibility of number from the Greek conception, governed by a natural intelligibility, to the modern conception governed by a symbolic intelligibility. He further suggests that the new science of the seventeenth century identifies, in a way necessarily hidden from itself, the symbolic mathematical representation of nature with nature itself. That is to say, annexed to the modern symbolic conception of number is a "symbolic nature" serving as the proximate object of modern mathematical physics. Indeed, Klein stresses, the algebraic form and conceptual structure of modern physics are inseparable:

> The symbolic language of algebra, that is, the language proper to mathematical physics, is not a purely technical or instrumental matter. It is a common mistake to believe that we can translate the theorems of mathematical physics into ordinary language, as if the mathematical apparatus used by the physicists were only a tool employed in expressing their theorems more easily.[8]

According to Klein, the point of departure for this algebraic physics, decisive for all subsequent science, is the "symbolic space" of Descartes' analytical geometry. Its ultimate legacy is the "symbolic unreality" of our modern civilization.[9]

In a thought-provoking essay on Klein and Husserl, Hopkins concludes that the symbolic character of modern mathematical science renders impossible in principle the fulfillment of Husserl's demand for the phenomenological reactivation or "cashing in" of its original intuitive evidence in the sensuous life-world. This is because the intentionality proper to that science terminates not in sensuous nature, but rather in symbolic mathematical entities themselves: "The consequence of this [formalization of meaning] is that the possibility of—however indirect—

an intuitive 'cashing in' of the formalized meaning formations of the *mathesis universalis* of modernity is *in principle* precluded."[10] If modern mathematical science cannot be "cashed in" intuitively in the sensuous life-world then its claim to knowledge of that world would seem to be radically called into question.

While there has been, by Hopkins and others, some valuable work done on the relationship between Klein's work on the history of mathematics and Husserl's philosophy of science, I believe that the notion of "symbolic nature" has yet to be truly brought into focus. This is, at least in part, because Klein did not himself do the corresponding work in the history of science, specifically on the algebraization of physics, that he did in the history of mathematics. Instead, we have some suggestive but nonetheless historically unsubstantiated remarks about Cartesian "symbolic space" serving as the "absolute space" of Newton's physics. My aim in this essay, accordingly, is to bring into focus the notion of "symbolic nature," which is barely suggested by Klein, with a view toward rendering more transparent the question of its possible phe-nomenological desedimentation. The architects of the new mathematical physics in the seventeenth and into the eighteenth centuries were in fact quite scrupulous about keeping distinct the symbolic quantities of algebra and the physical quantities of natural science. Indeed, symbolic mathematics in the form of algebraic equations does not become the dominant language of mathematical physics until about a century after Newton, and still it is not easy to fix the point at which it becomes an autonomous meaning formation as opposed to a mere shorthand for intuitively grounded knowledge of physical quantities based on the traditional mathematics of proportion. Klein's thesis, and with it the claim that modern mathematical physics ushers us into a world of "symbolic unreality," thus merits our most careful and critical consideration.

1. Symbolic Number

The central thesis of Klein's desedimentation of the history of modern mathematics in *Greek Mathematical Thought* can be summarized as follows: For Greek mathematics, a number is always a definite collection of countable units of a specific kind. In modern mathematics since Viète, by contrast, a number is essentially a symbolic entity defined by its general relationships to other numbers in a symbolic calculus.[11] For modern mathematics (to slightly modify a quip attributed to Quine), to be a number is to be the possible value of an algebraic variable. In Viète's reinterpretation of Diophantus' *Arithmetica*, Klein demonstrates, algebraic symbols undergo a change of teleology. While Diophantine algebraic symbols represent unknown quantities of count-

able units, Vièta's symbols represent the general concept of being a number. And this holds not just for algebraic variables themselves, but also for less general but still symbolic entities, such as "2," which directly refers no longer to two countable units but to "two-ness" in general or "the number two" as an object in its own right. Consequently, Klein points out, it is less than illuminating to merely characterize the modern symbolic conception of number as "abstract" in comparison to the "concrete" Greek conception.[12] Greek arithmetic, too, is a science of abstract number, at least if by "abstract" we mean a science capable of separating the general concept of number from particular kinds of countable units such as "three apples" or "three oranges." Nevertheless, Greek arithmetic never treats such general concepts as being numbers themselves.

In the scholastic terminology employed by Klein, Vièta takes a "second intention" *(intentio secunda),* or concept referring to another concept, and interprets it as a "first intention" *{intentio prima),* or concept applying directly to individual objects.[13] While this does not in itself institute the modern symbolic reification of number per se, as long as one keeps the original concept of number distinct from the symbolic, in the aftermath of Vièta's innovation—in Stevin, Descartes, and Wallis in particular—the original conception of number is implicitly replaced by the symbolic.[14] The implications of this "forgetfulness" for a mathematical science of nature are readily apparent.

The modern conception of number renders possible "numbers" impossible per se in Greek mathematics, such as "zero," "one," fractional numbers, negative numbers, irrational numbers, square-roots of negative numbers, and so forth. Of crucial importance, especially for subsequent mathematical physics, are fractional numbers, since in algebra and algebraic physics, ratios will be reinterpreted as fractions, and proportions as equality of fractional numbers. Moreover, the modern conception of number renders possible certain operations that in Greek mathematics would be incoherent. For example, Greek mathematics prohibits the multiplication of numbers by one another except in such case where the multiplication is interpreted as taking a certain number of units of length "by" another number of units of length to yield first an area ("two stades by two stades equals four stades square") and then a volume ("four stades square by two stades equals eight stades cube"). Similarly, in Greek mathematics one could not add together a square number and a cubic number, an operation we take for granted in modern algebra ($2^2+2^3=12$). This latter operation is rendered possible by the dimensionlessness and consequent homogeneity of symbolic number, which has been dissociated from direct reference to unit kinds.[15]

Clearly, from a phenomenological point of view the symbolic conception of number raises questions of intuitive fulfillment. In Husserlian terms, symbolic number is an ideal object which, as such, must find its fulfillment in "eidetic intuition." Eidetic intuitions, however, as *founded* acts of meaning fulfillment, are in principle genetically traceable to lower-level meaning intentions and their intuitive fulfillment, and ultimately to immediate experience of the life-world.[16] If the evidentiary genesis of such meaning formations cannot be reactivated, they "explode," so to speak, into incoherence. The problem raised by symbolic number, then, is whether there exists for it, to employ the terminology of Husserl's *Logical Investigations,* a corresponding "fulfilling sense."[17] Klein's analysis would seem to suggest not, for a fulfilling sense entails the possibility of the object being intuited as intended, and a second intention treated as a first intention cannot in principle be intuited *as intended.* Indeed, a second intention interpreted as a first intention would seem instead to be an example of what Husserl in *Logical Investigations* terms an "impossible meaning."[18]

However, even if no *direct* life-world fulfillment can be obtained for such symbolic entities, nothing precludes an indirect fulfillment if we reactivate the distinction between the original conception of number and the symbolic. For instance, I resolve to consume 14 more pieces of fruit per week and wish to determine how much I must increase my fruit intake per day. I set up the algebraic equation $7x=14$. Interpreted purely numerically (without reference to unit kinds), the expression finds fulfillment in eidetic intuition. To redeem the eidetic intuition in the life-world we restore the original units, in this case days, weeks, and fruit. While the product of a number of days and a number of fruits is intuitively incoherent, it can be rendered intuitive indirectly if we treat the formula $7x=14$ as an abbreviated proportion or ratio equation: "As seven days is to one day, fourteen pieces of fruit is to the number of pieces of fruit I should consume per day." Understood this way, symbolic meaning formations such as $7x=14$ in no way represent an insurmountable obstacle to Husserl's project of reactivating the sense-genesis of formalized meanings in the life-world.

Clearly, the matter is complicated, however, by the fact that symbolic number is not directly abstracted from countable units, but rela-tionally constituted as the possible value of an algebraic variable. Thus, in symbolic mathematics, "-4" qualifies as a number just as much as "4," since we have at our disposal an algebraic calculus defining operations on "-4" (and on negative numbers in general, e.g., negative times positive equals negative, negative times negative equals positive, and so forth). But what fulfilling sense, traceable to the life-world, could there be for negative numbers? One possibility often suggested is to

view negative numbers in terms of a departure from some neutral reference point, for instance, left instead of right on a number line, or owing money instead of having it. It is intuitively evident, for instance, that if I owe three dollars each to three people, I can multiply the "negative three" dollars ("negative" meaning it is a debt) by three, yielding a "negative nine" dollars. Here, the rule, "positive times negative equals negative," is intuitively evident. We can, perhaps, even cash in the algebraic rule, "negative times negative equals positive," by conceiving it as the repeated subtraction of a negative number.[19] Similarly, while the so-called "irrationals" cannot be cashed in as *numbers*, they can be redeemed as ratios of extensive magnitudes (ratios of line lengths, for instance).[20] On the other hand, such symbolic entities as square-roots of negative numbers ("imaginary numbers") appear representative of the type indicated by Hopkins, a categorial meaning formation that in principle cannot be reactivated, even indirectly, in terms of life-world experience. Nevertheless, theorems involving such numbers can be rigorously proven in algebra, evidently based on an eidetic intuition of the structural properties of the algebraic calculus itself. Symbolic mathematics, it seems, gives back more than we put into it in terms of life-world intuitions.

Klein notes that Greek mathematics distinguished between "logistic," the art of calculating, and arithmetic, the science of numbers. The latter dealt with such categories of number as even and odd, and so forth.[21] In logistic, by contrast to arithmetic, such things as fractional units could be used to facilitate calculation, even though they were not understood to be numbers. Diophantus' *Arithmetiea* springs from the soil of logistic and may be regarded as a kind of theoretical science of calculating.[22] Vièta's symbolic reinterpretation of Diophantine logistic is thus understood (by himself and others) in the sixteenth and seventeenth centuries as a calculational "art" in the sense of "logistic." But it is understood at the same time as the art of *finding the truth*, which is to say, a *universal science (mathesis universalis).* In this way, the symbolic method of representing the truth comes to be identified with science itself, and the proximate object of symbolic representation (symbolic meaning formations themselves) comes to be identified with the object of science.[23] Thus, the symbolic mathematics desedimented by Klein is emblematic of the "reification of method" reprehended by Husserl in *The Crisis* ("we take for *true being* what is actually a *method*"*).* But, if symbolic mathematics provides the decisive impetus for a general reification of method, it may be the case nonetheless that a nascent reification of method itself renders possible the emergence of symbolic mathematics. In this way, the otherwise paradoxical historical fact that modern "mathematical physics" does not actually become "mathematical," in the

sense we use the term today, until about a century after it has arrived on the scene with Newton's *Principia* could perhaps be made intelligible.

2. Symbolic Nature

Whatever implications the possible failure of fulfilling sense has for "pure mathematics" and its eidetic intuitions, such a failure can only appear problematic for a symbolic mathematical science of *nature* whose concepts, one should think, must be at least indirectly redeemable in the sensuous life-world if they are to be in any genuine sense "about" the natural world. As noted above, for Husserl, any authentic eidetic intuition must enjoy an evidentiary genesis traceable to immediate experience in the life-world. However, such evidentiary geneses assume differing forms for differing "regional ontologies." Pure mathematics deals with ideal meaning intentions (symbolic formulae) whose referents are themselves ideal objects. Consequently, intuitive fulfillment for such meaning intentions does not take the form of sensuous intuitions in the life-world, even though on Husserl's view their evidentiary genesis is, of necessity, traceable to life-world intuitions. Such ideal objects appear "in person," as it were, to eidetic intuition itself.

However, while the objects of pure mathematics are *ideal,* the objects of mathematical physics are *real.* Thus, for the symbolic meaning formations of mathematical physics to have a fulfilling sense means, on Husserl's assumption, that they enjoy at least indirectly an intuitive fulfillment in the sensuous life-world. Einstein's $E=mc^2$, for instance—a symbolic meaning formation whose object is not directly intuitable in the sensuous life-world—must on Husserl's account either have an indirect and sensuously intuitable non-symbolic referent, as in our earlier example of the fruit, or explode into incoherence as an impossible meaning, something on the level of "three apples times four oranges equals twelve apple-oranges."

While the Husserl of *The Crisis* is clearly troubled by the obscurity of the connection between symbolic mathematical physics and the sensuous life-world, he does not question the authenticity of mathematical physics itself as a meaning formation.[24] That is to say, Husserl never suggests that, for instance, Einstein's theory of relativity might be an "impossible meaning" which would explode into incoherence were we to desediment its ideal sense-history. His concern is, rather, *how* meaning accrues to such symbolic formations via their genesis in the intuitively given life-world, not *whether* it so accrues:

> Einstein's revolutionary innovations concern the formulae through which the idealized and naïvely objectified *physis* is dealt with. But how formulae in general, how mathematical objectification in general, receive meaning on the foundation of life and the intuitively given surrounding world—of this we learn nothing; and thus Einstein does not reform the space and time in which our vital life runs its course.[25]

Thus, as Hopkins observes, Husserl does not *argue* for this genesis of symbolic mathematical physics in the life-world, but simply sets forth the disclosure of that genesis as a phenomenological task.[26] Because life-world intuition is the means by which the world discloses itself originally, any representation of the world that has become "unteth-ered," as it were, from life-world intuition is thereby nullified. This need not preclude inferred theoretical entities, such as gravitational fields and such, just so long as the sense genesis of such entities is rendered intuitively transparent at every step.

A second desedimentation by Klein, however—this one focusing on Descartes' interpretation of the geometrical figures of his unpublished *Regulae* (1619-1628) and then *Geometry* (1637)—would appear to call that phenomenological task into question.[27] Descartes' analytical geometry, as Klein brilliantly demonstrates, sets up a "symbolic space" quite distinct from the real space of traditional geometry. Descartes' figures, principally line lengths but also geometrical "figures" in the more usual sense, are not *figures in space* per se, but rather symbolic representations of "magnitudes in general."[28] Indeed, it is demonstrable that these "magnitudes in general" are *symbolic numbers* in Klein's sense. Descartes launches his *Geometry*, for example, with the assertion that "[a]ny problem in geometry can easily be reduced to such terms that a knowledge of the lengths of certain straight lines is sufficient for its construction."[29] He proceeds by setting forth the method for multiplying two lines together via a geometrical construction exhibiting proportions between line segments. Further noting that we can economically represent the lengths of the line segments with letters, Descartes illustrates the latter procedure with the expression a^2 or "*a* multiplied by itself; by "square" and other such expressions, he remarks, he really means "simple lines, which, however, I name squares, cubes, etc., so that I may make use of the terms employed in algebra."[30] Since multiplication of lines to yield lines is an intuitively incoherent operation, we can conclude that Descartes is multiplying *symbolic* (and therefore dimension-less) *numbers*, themselves symbolized now by letters, now by lines, and then symbolizing the product itself with a line or a letter. The figures of Descartes' analytic geometry, then, are intuitively perspicacious symbolic representations of general relationships between magnitudes—

relationships which can be more economically, albeit less perspica-ciously, symbolized by letters in an algebraic equation. For instance, when we "graph" the equation for a circle ($x^2+y^2=r^2$), the resulting figure is itself circular, but it is not intended as a circle per se, but rather as a symbolic representation of the general quantitative relationships expressed by the equation for a circle. These quantitative relationships are not specific to geometrical circles. Thus, the Cartesian figure (fig. 1) is only indirectly a representation of a circle in space, whereas a Euclidean figure (fig. 2) drawn with a compass, for instance, is a direct representation:

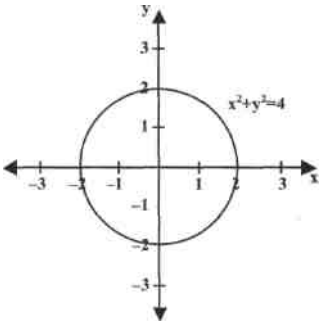

Figure 1

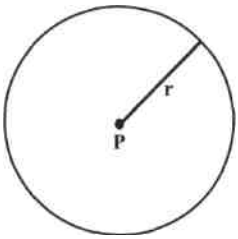

Figure 2

The point can perhaps be rendered more clearly by an example where the symbolic figure does not resemble that which it represents. We can represent, for instance, the relationship between time and distance in uniformly accelerated, straight-line motion with the equation $s=at^2/2$, which graphs as a parabola (x-axis represents distance, y-axis represents time):

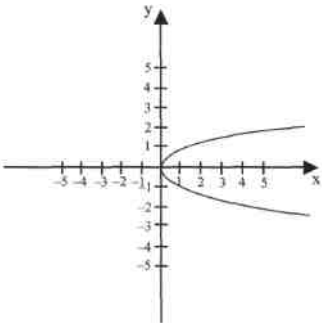

Figure 3

This parabola does not resemble the straight-line motion it represents (indeed, the portion beneath the x-axis does not represent anything in the motion at all), and bears exactly the same relationship to that motion as does a Cartesian symbolic circle to the circle in space represented by it.

However, Klein further suggests that Descartes has implicitly *identified* the "symbolic space" of his analytical geometry with real space, such that Cartesian symbolic space becomes the "absolute space" of Newtonian mechanics.[31] As John Schuster notes, this interpretation seems to overreach the text, in which the symbolic *employment* of real extension does not in itself constitute an *identification* of "symbolic space" and real space.[32] To be sure, Descartes does not seem to distinguish between his symbolic figures and the properly geometrical figures of traditional constructive geometry. Descartes' symbolic figures, after all, happen to be instances of the very geometrical figures they represent symbolically. That is, his symbolic circle is in fact circular, his symbolic ellipse is in fact elliptical, and so forth. But even if Descartes does in fact identify the symbolic space of his geometry with physical space—and this seems to be a conjecture on Klein's part— Klein adduces no specific evidence from "Newtonian science" itself to back up the claim that the latter is somehow essentially dependent upon Cartesian geometry.[33] Such a claim, however suggestive, would have to be cashed in historically itself.

3. Mathematical Physics and the Life-World

It will be helpful for elaborating the concept of "symbolic nature" and its relationship to life-world intuition to briefly consider some key features of the historical process by which algebra was adopted as the language of modern physics. This was a slow development and met with significant resistance. Some of the reasons for the delay are clear, having to do with the felt need among physicists to keep symbolic and physical quantities conceptually distinct. One would naturally expect Descartes, for instance, to employ his newly developed "analytical geometry" in mechanics, which latter is after all the defining telos of Descartes' *mathesis universalis.* But, in fact, we find no algebraic equations at all in Descartes' *Principles of Philosophy,* the definitive statement of his mature physics. While the generally non-quantitative character of Descartes' physics has often been remarked and various reasons for it given, one might nevertheless ask why Descartes does not at least employ algebra in the formulation of his clearly quantitative laws of nature and rules of impact. Descartes describes his groundbreaking law

of conservation of quantity of motion, for instance, in the following terms:

> In the beginning . . . [God] created matter, along with its motion and rest; and now, merely by his regular concurrence, he preserves the same amount of motion and rest in the material universe as he put there in the beginning. . . . Thus if one part of matter moves twice as fast as another which is twice as large, we must consider that there is the same quantity of motion in each part; and if one part slows down, we must suppose that some other part of equal size speeds up by the same amount.[34]

Here, Descartes expresses "quantity of motion" as a traditional compound ratio—a body's quantity of motion is jointly proportional to its speed and its size or volume. Why does he not simply write, as we would, $Q=VS$ (Q=quantity of motion, V=volume, S=speed), and then express the conservation law for two colliding bodies as $V^1S^1_0+V^2S^2_0= V^1S^1_f+V^2S^2_f$ (the subscripts designate initial and final velocity before and after the collision; and the superscripts distinguish the two bodies)? Perhaps the quantitative relationships are immediately evident in this case, obviating the need for symbolic technique. Beyond that, however, another and more serious impediment presents itself, namely, that speed and volume, as non-homogeneous quantities, cannot be multiplied together.[35] Thus, Descartes employs the language of proportion, because, along with everyone else in the seventeenth century, he understandably regards the language of proportion as the language of mathematical physics.

John Wallis in 1685 still prohibits algebraic ratios between non-homogeneous magnitudes in physics, although he later admits them as abbreviated ratio equations (proportions expressed as an equality of fractional ratios), not as "algebraic equations" in the sense we understand the term today.[36] Since length and weight are heterogeneous quantities, Wallis notes, they have no ratio. Similarly, velocity is not expressed algebraically as a ratio of distance and time—something we take for granted today—until Varignon around 1699.[37] It is not until the third edition of the *Principia* (1726) that Newton, somewhat reluctantly, expresses quantity of motion (a vector quantity for him in the sense of our "momentum" or *mv)* as the product, "if I may so say," of mass and velocity.[38] Still he refrains from the algebraic expression *mv.* Moreover, even when such algebraic expressions come into common usage in the eighteenth century, they are understood in general as abbreviated compound ratios, not absolute quantities in their own right. Finally, when Laplace, for instance, in his *Traité de mécanique céleste* (1798), at last argues for an absolute interpretation of the equations of mathematical physics, he feels the need to stress that such

equations express homogenous ratios between "abstract" or dimension-less (read, Klein's "symbolic") numbers rather than direct ratios between inhomogeneous quantities:

> Time and space, being heterogeneous quantities, cannot be *directly* compared with each other; therefore an interval of time, such as a *second* is taken for the *unit* of time and a given space, such as a *metre,* is taken for the unit of space; then space and time are expressed by abstract numbers, denoting how many measures of these particular species each of them contains, and they may then be compared with each other. In this manner the velocity is expressed by the ratio of two abstract numbers, and its *unit* is the velocity of a body which describes one metre in a second.[39]

Laplace here interprets the equation $v=d/t$ (or, in differential calculus, $v=ds/dt$) as a pure ratio of symbolic numbers expressed as a fraction, which is then "plugged back into" the units of velocity. The passage clearly implies that the equation can be cashed in intuitively by the following translation: "The ratio of velocity to its unit is proportional to the compound ratio of the distance traversed to its unit and the unit of time to the time elapsed." If there is the substitution of a symbolic ideality *(ds/dt)* for a physical reality (velocity) here, it is hardly a surreptitious one!

A number of points may be noted in these examples. First, a significant resistance, stemming from the desire to keep symbolic and physical quantities distinct, had to be overcome in the introduction of an algebraic language into physics. Second, even when they were finally accepted into physics, for a century or more, algebraic equations were interpreted as abbreviated proportions rather than as absolute equalities in the sense we understand the equations of physics today. Third, until at least the beginning of the nineteenth century, algebraic equations, even when they were interpreted as absolute equalities rather than as abbreviated proportions, were explicitly understood as translatable back into the language of proportion and redeemable in intuitive experience. Clearly, the mere usage of algebra in mathematical physics is not tantamount to a symbolic reification of nature, even when it yields symbolic entities, such as mv or v/t, containing sedimented operations lacking for any life-world fulfilling sense. It would be in general prohibitively cumbersome to intuitively cash in the algebraic formulae of modern physics. I do not know the extent to which it has been tried. Certainly, the training of scientists neither includes such "cashing in" as part of the curriculum nor encourages it, and this no doubt fosters the interpretation of symbolic entities as physical entities in their own right. One speaks of energy, for instance, as "being" mc^2, without thinking of that expression as shorthand for a complex proportion of intu-

itable ratios among physical quantities. Nevertheless, one should think that the equations of modern mathematical physics are at least in principle indirectly redeemable in intuition. This turns out not to be the case, as an example will demonstrate.

In Hermann Minkowski's 1908 formulation of four-dimensional "space-time" for Einstein's special theory of relativity, a physical quantity later designated the "space-time interval" is introduced. We first introduce a space-time coordinate system ("Minkowski diagram") in which time and space are represented respectively on the vertical and horizontal axes of a Cartesian coordinate plane. Each point in the diagram represents an "event" in space-time, with its proper space *(x)* and time (y) coordinates. A locus of points representing successive events (positions) associated with a body in motion designates that body's "world line" (fig. 4).[40]

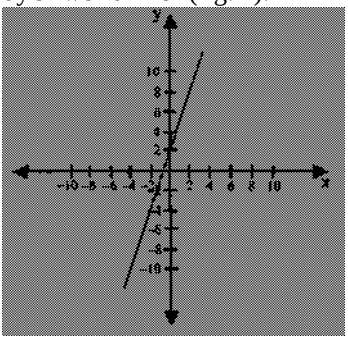

Figure 4

Since Einstein's "principle of relativity" dictates that the laws of nature be the same for all coordinate systems, regardless of their motion relative to one another,[41] his original formulation of the special theory of relativity contains a set of equations (the "Lorentz transformation") on the basis of which may be derived the coordinates for an event in one coordinate system based on the coordinates for that same event in another coordinate system in motion relative to the first. It turns out that for any two events, the invariant quantity $c^2f—x^2$ (c=velocity of light, t=time, x=distance) is defined for all coordinate systems. This quantity mathematically resembles the invariant distance x^2+y^2 between two points in space for a rotation of coordinate axes (see fig. 5), suggesting the possibility that the Lorentz transformation might itself be a kind of "rotation of axes" in a four-dimensional "space-time." In that case, we could speak of the "space-time interval" (s^2) between two events.

It turns out that the analogous rotation of axes can be performed and the invariant space-time interval obtained, if we employ a "hyperbolic" trigonometry in which angles are measured on arcs of a hyperbola (as opposed to arcs of a circle as in regular trigonometry):

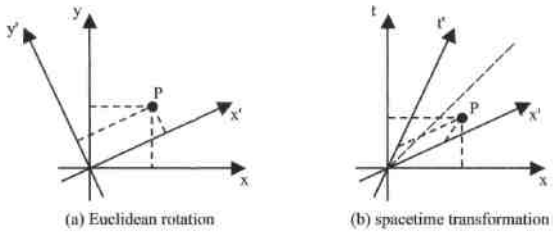

Figure 5. In the diagram on the left, the distance between the origin and point P clearly remains invariant when the coordinate axes are rotated through an angle. In the "space-time" rotation on the right, which represents a change from one coordinate system to another in motion relative to the first, the space-time interval remains the same. Note, however, that while the invariant distance in (a) is the distance between the origin and point *P*, this is not the case for the invariant space-time interval in (b).

What is the link between the invariant c^2t^2-x^2 and the physical world, such that the former can be interpreted as a really existing space-time interval as opposed to being merely a mathematical artifact? Clearly the quantity c^2t^2-x^2 cannot itself be a space-time interval since it has units of distance.[42] And it is of no avail to assert that since c^2t^2 expresses time in units of distance, we can interpret c^2t^2-x^2 as an actually existing space-time interval. For since c^2t^2 is merely a spatial magnitude being used to *represent* time, this would indeed be to confuse the means of representation with the thing represented! It is customary, therefore, to regard the velocity of light as unity, and to substitute for c the symbolic and dimensionless number "1." The expression c^2t^2-x^2 is by this means transformed to τ-x^2 (where τ=1*t*). But what now renders possible arithmetically the subtraction of x^2 (an interval of distance) from the heterogeneous τ^2 (an interval of time)? Only symbolic mathematics, which allows us to perform the operation on two dimensionless numbers and then plug the result back into the units of space-time. And, by contrast with our previous examples, in this case there is no way of interpreting τ^2-x^2 as symbolic shorthand for something intu-itable in the sensuous life-world. The space-time interval is an irre-ducibly symbolic entity.[43]

What *justifies* the non-intuitive operation τ^2-x^2 scientifically? Simply this, that measured times and distances in the life-world, contingent as they are on an arbitrary choice of coordinate systems, cannot represent

the "real world." Thus, space and time of necessity constitute a homogeneous four-dimensional continuum, notwithstanding the fact that as phenomena in the life-world they are irreducibly heterogeneous.[44] Consequently, the space-time interval, an irreducibly symbolic entity seemingly enjoying no intuitive sense in principle, is the truly real, while times and distances experienced in the life-world by actual observers are mere appearances. Husserl's "life-world," the "only real world," is now precisely the unreal world, and symbolic space-time is now the truly real world.[45] Thus, Minkowski concludes that the postulate of relativity, according to which "only the four-dimensional world in space and time is *given by phenomena* [my emphasis]" could be more appropriately termed the *"postulate of the absolute world* [Minkowski's emphasis]."[46] One could hardly devise a more complete reversal of Husserl's phenomenology of the life-world.

It is worth quoting, to conclude our discussion of this example, physicist David Bohm's remarks on the conceptualization of the world inherent in Minkowski's approach: "In the procedure described above [the Minkowski diagram], *the analysis of the world into constituent objects has been replaced by its analysis in terms of events and processes, organized,* ordered, and structured so as to correspond to the characteristics of the material system that is being studied."[47] Corporeal beings themselves, in other words, are to be replaced by "events" defined in terms of "order" within a symbolic calculus, this symbolic representation "corresponding to" rather than directly representing the physical world. In this *systematic* interpretation of space-time, we have the exact counterpart in mathematical physics of the symbolic representation of number inaugurated by Vièta. Rendered systematically, the real entity becomes a nodal point or terminus, as it were, in a nexus of relations determined by the method of representation.

4. Conclusion

Arthur Stanley Eddington famously remarked upon the "doubling" of the life-world by the world of mathematical physics. Yet, to a significant extent, our life-world is, as Klein suggests, already itself a symbolically reified world:

> These features of the *mathesis universalis,* which appear most forcefully in our Science of Nature and dominate our entire manner of thinking, can, I trust, be traced in the social and economic fields in which we live. Along the lines of our society, every one of us must "do his job" according to certain rules imposed on us by ever-working machineries. The production and consumption of goods have acquired a sort of "automatic" character. No one can escape

> the fatality which is the result of this automation. Our life, then, even our most intimate life, is completely conditioned by social and economic necessities which are alien to ourselves and which we nevertheless accept as the true expression of ourselves. Our work, our pleasures, even our love and our hatred are dominated by these all-pervading forces which are beyond our control.[48]

Only this, it seems to me, can explain the propensity of scientists to project their symbolic constructions onto the life-world as if the latter existed in seamless continuity with the symbolic mathematical world. It is not so uncommon, for instance, to come upon claims such as that, since time has no independent standing in modern physics, the perceived "flow" of time is an illusion. Physicist Paul Davies, a prolific writer of popular books on science, suggests that, if we were to think more relativistically and "pin down those brain processes" that give us the illusion of the passage of time, we could rid ourselves of the fear of death.[49] But in reality, thinking relativistically can have no effect whatsoever on our life-world experience of time, since in the life-world, time is always and essentially distinct from space. And it is not true, as is often suggested, that this is merely because we lack the experience of traveling at velocities appreciably large in comparison with the velocity of light. Were we to experience such velocities, we would also experience a number of relativistic effects we are not used to experiencing, such as the motion-dependency of clock rates, mass, and so forth. But we would still experience the world from a specific frame of reference, never from the symbolic perspective of "space-time," and time would remain fundamentally distinct from space. There is a poignant anecdote about Einstein trying to console himself on the death of his friend Michele Besso by calling to mind the irreality of the flow of time in the theory of relativity. Perhaps it is true that "space-time" gestures toward a transcendent reality in which things are not subject to the passage of time: that, as Augustine suggests, eternity upholds time, whose parts—past, present, and future—are all forms of non-being.[50] Needless to say, this alters nothing about the reality of time and death in the life-world. The idea that it does is symptomatic of the "symbolic unreality" (Klein) in which we live.

Husserl's project of desedimentation presupposes that any concept of physics that lacks at least an indirect *life-world* fulfilling sense is devoid, *ipso facto,* of any valid sense at all. However, precisely the type of historical investigation recommended by Husserl's *Crisis* calls this supposition into question.[51] For modern mathematical science represents its object (the natural world) through a form of eidetic intuition terminating in its own symbolic structure. It cannot connect to the sensuous life-world via corresponding intuitions in the life-world, but

rather solely via a *correlation* between its eidetic intuitions (symbolic mathematical formulae) and sensuous intuitions in the life-world (experimental observations). These life-world, experimental observations cannot in general be regarded (and are not so regarded by scientists themselves) as intuitive fulfillments of the symbolic meaning intentions of mathematical physics, since they do not intuit the objects of mathematical physics *as those objects are intended,* but rather merely correspond to those meaning intentions. The experimental evidence for "space-time" comprises, for instance, tracks on a photographic plate exposed to high energy particles and the like. Space-time itself simply cannot make an appearance, even indirectly, to life-world intuition.

In the context of Husserlian phenomenology, the question must consequently arise as to whether the concepts and symbolic formulae of mathematical physics are thereby rendered inauthentic—whether mathematical physics, that is, finally deals in *impossible meanings* (such as "space-time"). In that event, modern mathematical physics would simply be a powerful tool for making predictions and manipulating nature technologically, but not a *science* in Husserl's sense. While Husserl does not in general seem to have countenanced the possibility, his commitment to the life-world as the "only real world" must inevitably lead to the conclusion that symbolic mathematical physics is not science per se, since, while the regional ontology governing symbolic mathematical physics is one of *real* rather than *ideal* objects, its symbolic meaning formations are nevertheless irredeemable in the life-world. While an adequate treatment of whether it is the right conclusion is beyond the scope of this essay, the weight of our analysis, it seems to me, points the other way. Certainly the issue cannot be decided by an *a priori* commitment to the life-world inimical to the very spirit of transcendental phenomenology. To be sure, at the level of life-world experience, the concept of "space-time" is incoherent, projecting as it does a mathematical homogeneity on space and time when they are experien-tially heterogeneous. However, mathematical physics does not merely generate symbolic formulae and test their predictions experimentally. Meaning accrues to these very formulae via the conceptual structure of theories. In our example, the conceptual structure of the theory of relativity reveals an incoherence in our life-world concepts of space and time. For while the intuited heterogeneity of space and time logically implies the existence of a privileged frame of reference for which alone the laws of nature are valid, the concept of such a privileged frame of reference appears itself to harbor a sedimented incoherence. From a phenomenological perspective, then, it would appear that *the life-world itself embodies impossible meanings.*

Mathematical physics consequently is led to construct a symbolic realm of meaning, transcending the life-world. Indeed, in some mysterious way, nature seems to make an appearance "in person" through this symbolic realm, the latter accessible only to a mathematical-symbolic form of eidetic intuition and in principle hidden from sensuous experience in the life-world. The symbolic reification of method appears even to enjoy, in the words of Minkowski, a kind of "pre-established harmony" with nature itself.[52] The irony of the situation is that, perplexed as he is in *The Crisis* about how the initial modern impulse to "rigorous science" becomes derailed, so to speak, in historically given science, Husserl evidently fails to see that the very demand for a transparent evidentiary foundation in the life-world for science may be regarded as a reification of the phenomenological method itself. In this connection, Hans Blumenberg, commenting on Husserl's interpretation of Galileo, rightfully observes that, "[a]s a philosopher of history . . . [Husserl] remained the Cartesian he had always been."[53] For what ensures that the world gives itself transparently in direct intuition of the life-world, as prescribed by Husserlian phenomenology?

Our modest attempt at desedimenting "space-time" suggests that the life-world cannot function as a horizon for meaning in the way Husserl desires, because it is a *self-transcending* horizon for meaning. The concept of "space-time," after all, has its very genesis in the attempt to render *experienced* time *intuitively consistent with itself* by defining the physical conditions for the possibility of its objective determination.[54] Symbolic mathematical physics, indeed, finally seems to transcend its own historical genesis as reification of method, comes bearing gifts of unanticipated beauty and grace. So, at least, describes the physicist Heinrich Hertz regarding Maxwell's electromagnetic field equations: "It is impossible to study this wonderful theory without feeling as if the mathematical equations had an independent life and intelligence of their own, as if they were wiser than ourselves, indeed wiser than their discoverer, as if they gave forth more than he had put into them."[55] If such a thought has any merit, then at least one task of historical phenomenology of science is to distinguish such genuine gifts from the thoughtless reification of symbolic method reprehended by Husserl. Symbolic mathematical physics, it would seem, is indeed one path to that "only real world" that must finally condition the life-world itself.

NOTES

1. Edmund Husserl, *The Crisis of European Sciences and Transcendental Phenomenology*, trans. David Carr (Evanston: Northwestern University Press, 1970).

2. Ibid., p. 58.

3. Ibid., pp. 49-50.

4. As is well-known, Husserl's account of the origin of geometry appeals to the tradition of practical measurement as an infinitely perfectible art yielding idealized "limit-shapes" (see ibid., pp. 24-8, 375-8). This process of idealization, scientifically realized in Euclidean geometry and sedimented in the received geometrical tradition, is identified unawares by Galileo with physical body itself. Thus, Galilean science is simultaneously revealed in its original intuitive evidence (the perception of empirical shapes) and exposed as a "surreptitious substitution" of idealities for the real world. Husserl's interpretation of the origin of geometry has been criticized and seems questionable in a number of respects. It is not clear, for instance, that the origin of geometry lies in the kind of progressive approximation to limit-shapes through practical measuring that Husserl describes. Patrick Heelan argues that the scientific practice of measurement assumes no such limiting processes, being rather governed by pragmatic considerations of appropriateness in a given context (see Patrick Heelan, "Husserl's Later Philosophy of Natural Science," *Philosophy of Science* 54:3 (1987), pp. 368-90). Moreover, a more "Platonic" account, if you will, would see in the very teleology of approximation a forehaving or "recollection," as it were, of such geometrical idealities themselves. On such a view, the geometrical idealization of nature involves no necessary substitution of the ideal for the real, since bodies appearing to sense perception are constituted already in their very being by a participation (-------) in ideal mathematical forms (Husserl mentions this possibility in passing but does not pursue it; see Husserl, *The Crisis*, p. 23). There is in fact some evidence that Galileo is indeed "Platonic" in at least the sense we have indicated. In the *Dialogue Concerning the Two Chief World Systems*, for instance, Galileo appeals to the Platonic doctrine of recollection ("nostrum scire sit quoddam reminisce"; see Galileo Galilei, *Dialogue Concerning the Two Chief World Systems—Ptolemaic & Copernican*, trans. Stillman Drake, 2nd rev. ed. [Berkeley: University of California Press, 1967], pp. 90-1). The question of Galileo's "Platonism" is beyond the scope of this essay, but it has received extensive discussion among scholars. Suffice it to say that, while from the perspective of his conviction that the corporeal world is the proper object of genuinely scientific knowledge, Galileo clearly is no "Platonist," he is, nonetheless, in some sense Platonic, or perhaps, better, Pythagorean, in his conviction that mathematics governs the intelligibility of the physical cosmos. However, in the context of Husserl's enterprise, such categorizations are wide of the mark, since what Husserl decries in Galilean science is something constitutive of a distinctly modern outlook. For some helpful discussions along with references to literature on the issue of Galileo and Platonism, see, for example, Hans Blumenberg, *The Genesis of the Copernican World*, trans. Robert M. Wallace (Cambridge: MIT Press, 1987), pp. 410-9; Dominique Dubarle, "Galileo's Methodology of Natural Science," in *Galileo: Man of Science*, ed. Ernan McMullin (New York: Basic Books, 1967), pp. 295-314; Thomas P. McTighue, "Galileo's 'Platonism': A Reconsideration," in *Galileo: Man of Science*, pp. 365-87; and Ernst Cassirer, who addresses the issue in several

works, for example, "Mathematical Mysticism and Mathematical Science," trans. Ernan McMullin, in *Galileo: Man of Science,* pp. 338-51.

5. Husserl, *The Crisis,* pp. 22, 8.

6. Ibid., p. 51.

7. Klein's seminal work on the theme is *Greek Mathematical Thought and the Origin of Algebra,* trans. Eva Brann (Cambridge: MIT Press, 1968). It is based on work carried out in the early 1930s, and, thus, actually predates Husserl's *Crisis* (see Burt C. Hopkins, "Crisis, History, and Husserl's Phenomenological Project of Desedimenting the Formalization of Meaning: Jacob Klein's Contribution," *Graduate Faculty Philosophy Journal* 24:1 [2003], p. 89). There is a number of later essays by Klein bearing on the subject collected in *Jacob Klein: Lectures and Essays,* ed. Robert B. Williamson and Elliot Zuckerman (Annapolis: St. John's College Press, 1985), the most important of which is "The World of Physics and the 'Natural' World" (pp. 1-34). Thomas Ryckman, in *The Reign of Relativity: Philosophy in Physics 1915-1925* (Oxford: Oxford University Press, 2005), has recently highlighted the work of Hermann Weyl, who under the influence of Husserl's transcendental phenomenology, attempted a form of phenomenological reconstruction of the general theory of relativity, during the period between 1918-23. I will not be addressing Weyl in the present essay.

8. Klein, "Modern Rationalism," in *Lectures and Essays,* p. 61.

9. Ibid., p. 64.

10. Hopkins, "Crisis, History, and Husserl's Phenomenological Project," p. 100. For a similar view, see Ian Angus, "Jacob Klein's Revision of Husserl's *Crisis: A* Contribution to the Transcendental History of Reification," *Philosophy Today* 49:5 (2005), pp. 204-11.

11. For a general treatment of Klein's interpretation in *Greek Mathematical Thought,* see Burt C. Hopkins, "Jacob Klein on François Viète's Establishment of Algebra as the General Analytical Art," *Graduate Faculty Philosophy Journal* 25:2 (2004), pp. 51-85.

12. Klein, *Greek Mathematical Thought,* pp. 175-6.

13. Ibid., pp. 174-5. Klein's account in "The World of Physics and the 'Natural' World," in *Lectures and Essays,* pp. 17-21, 24-6, *et passim,* is somewhat easier to follow.

14. See Klein, "Phenomenology and the History of Science," in *Lectures and Essays,* p. 81. It is worth pointing out that the original conception of number still echoes in our everyday speech, for example, when we say such things as, "I have a number of friends coming over tonight" (a good joke on one's spouse if the number is zero).

15. While Viète himself still attempts to observe a "rule of homogeneity" in his algebra, the symbolic conception of number in principle renders it irrelevant. Descartes, in his *Geometry,* for instance, treats both numbers and symbolic line lengths as dimensionless entities.

16. Husserl, in *Formal and Transcendental Logic,* speaks of the "non-predicative" evidence of non-categorially mediated experience: "Accordingly, from these genetical points of view, *the intrinsically first judgment-theory is the theory of evident judgments, and the intrinsically first thing in the theory of evident judgments* (and therefore in judgment-theory as a whole) *is the genetical tracing of predicative evidences back to the non-predicative evidence* called *experience"* (Edmund Husserl, *Formal and Transcendental Logic,* trans. Dorion Cairns [The Hague: Martinus Nijhoff, 1978], p. 209).

17. The "fulfilling sense" is the "object's ideal correlate in the acts of meaning-fulfillment which constitute it" (more specifically, the "ideal unity" of its possible meaning fulfillments) (Husserl, *Logical Investigations,* vol. 1, trans. J.N. Findlay [London: Routledge and Kegan Paul, 1970], p. 290). For instance, the empty intention of an automobile is fulfilled when the automobile itself is encountered intuitively (by seeing it in the driveway, and so forth). However, the one selfsame automobile implies an infinite horizon of possible meaning fulfillments (it can be seen from various angles, for example), the ideal unity of which is its "fulfilling sense."

18. In *Logical Investigations,* pp. 291-5, Husserl defines "impossible meanings" as meaning intentions that in principle lack a fulfilling sense. The genetic treatment demanded by this notion, suggesting as it does sedi-mented incoherencies, comes only later in Husserl's writings, especially *Formal and Transcendental Logic.*

19. In other words, just as 3 times 2 can be understood as 0+2+2+2=6, -3 times -2 can perhaps be cashed in as 0-(-2)(-2)-(-2)=6.

20. "Irrational" numbers are numbers that cannot be expressed as whole number fractions (or "ratios" in the modern sense of the term).

21. See, for instance, Klein, "The opposition of logistic and arithmetic in the Neoplatonists," chap. 2 of *Greek Mathematical Thought,* pp. 10-6.

22. See Klein, "The *Arithmetic* of Diophantus as theoretical logistic. The concept of *eidos* in Diophantus," chap. 10 of *Greek Mathematical Thought,* pp. 126-49.

23. See Klein, "Modern Rationalism," pp. 59-60.

24. An "authentic" meaning is for Husserl simply one for which there exists a fulfilling sense.

25. Husserl, *The Crisis,* p. 295.

26. Hopkins, "Crisis, History, and Husserl's Phenomenological Project," p. 82.

27. Klein's discussion of Descartes' geometry can be found in *Greek Mathematical Thought,* pp. 197-211; and "The World of Physics and the 'Natural' World," pp. 12-21.

28. "It is easy to conclude from this that it will be very useful if we transfer what we understand to hold for magnitudes in general to that species of magnitude which is most readily and distinctly depicted in our imagination. But it follows from what we said . . . that this species is the real extension of a body considered in abstraction from everything else about it

save its having a shape" (René Descartes, *Rules for the Direction of the Mind,* in *The Philosophical Writings of Descartes,* vol. 1 trans. John Cottingham, Robert Stoothoff, and Dugald Murdoch [Cambridge: Cambridge University Press, 1985], p. 58).

29. René Descartes, *The Geometry of René Descartes,* trans. David Eugene Smith and Marcia L. Latham (New York: Dover, 1954), p. 2.

30. Ibid., p. 5.

31. See Klein, *Greek Mathematical Thought,* pp. 210-1; and "The World of Physics and the 'Natural' World," p. 21. It is not clear the extent to which Klein wishes to claim that Cartesian symbolic space grounds the physics of Newton himself as opposed to a more generally conceived "Newtonian physics." In *Greek Mathematical Thought,* p. 211, he says that Cartesian symbolic space is the "foundation on which Newton will raise the structure of his mathematical science of nature," while in the later essay, p. 21, he refers simply to "Newtonian physics."

32. John A. Schuster, "Descartes' *Mathesis Universalis:* 1619-28," in *Descartes: Philosophy, Mathematics and Physics,* ed. Stephen Gaukroger (Sussex: Harvester Press, 1980), p. 193n.l29. In Descartes' *Regulae,* the symbolic employment of extension is clearly in the interest *of legitimating* universal mathematics, the object of which is identical to the object of Descartes' projected "physico-mathematics." That is to say, the identification of the "real extension" functioning symbolically in the *mathesis universalis* of rule fourteen, the science of "general magnitude" or "order and measure," and the extension indirectly impressed by external objects upon the corporeal imagination in the mechanistic account of perception set forth in rule twelve, certify "physico-mathematics" as a genuine science of the corporeal world. While the legitimatory program of the *Regulae* is superseded in later writings, especially the *Meditations,* the imagination must still play some legitimatory role, since it alone can serve as medium for the transmission of sense information regarding the actual shapes of external bodies. On the reasons for the eclipse of the legitimatory program of the *Regulae,* see ibid., pp. 73-9. Klein notes the decline of the legitimatory regime of the *Regulae* in Descartes' later writings, but insists that the symbolic conception of actual space remains "essentially untouched" by this later development (see Klein, *Greek Mathematical Thought,* pp. 308-9n. 328).

33. It has in fact recently been argued that the first lines of Newton's preface to the *Principia,* in which geometry is said to be "founded on mechanical practice" *(Mathematical Principles of Natural Philosophy,* trans. Andrew Motte, rev. ed., Florian Cajori [Berkeley: University of California Press, 1934], p. xvii), are directed against Descartes' algebraic method as laid out in the *Geometry*—and for reasons having to do with the latter's "constructive" character or perceived lack of intuitive content (see Mary Domski, "The Constructible and the Intelligible in Newton's Philosophy of Geometry," *Philosophy of Science* 70:5 [2003], pp. 1114-24; and Niccolo Guicciardini, "Geometry and Mechanics in the Preface to Newton's *Principia:* A Criticism of Descartes' *Géométrie," Graduate Faculty Philosophy Journal* 25:2 [2004], pp. 119-59). Such a polemic does not, of

course, preclude the possibility of Newton having "internalized" unawares, as it were, Cartesian symbolic space. But once again, such a claim requires specific historical evidence.

34. Descartes, *Principles of Philosophy*, in *The Philosophical Writings of Descartes*, vol. 1, p. 240.

35. This is not quite precise, since not even homogeneous quantities can be multiplied intuitively, multiplication being a process of repeated addition of a quantity to itself. In our present-day algebra, we multiply dimension-less symbolic numbers.

36. See John Wallis, *A Treatise of Algebra, Both Historical and Practical* (London, 1685), pp. 134-6; and John J. Roche, *The Mathematics of Measurement: A Critical History* (London: The Athlone Press, 1998), p. 87.

37. Ibid., p. 102.

38. Isaac Newton, *Mathematical Principles of Natural Philosophy*, trans. Andrew Motte, rev. ed. by Florian Cajori (Berkeley: University of California Press, 1934), pp. 23, 5.

39. Pierre Simon de Laplace, *Traité de mécanique céleste*, vol. 1 (Paris: Imprimerie Royale, 1843-6), p. 15, cited in John J. Roche, *The Mathematics of Measurement* (London: The Athlone Press, 1998), p. 138.

40. As is customary, for convenience we include here only one spatial dimension.

41. It follows from the relativity of motion that no particular frame of reference is privileged over another. In the special theory of relativity, the principle of relativity is limited to "inertial" or unaccelerated reference frames.

42. This follows from the fact that the product of velocity and time is distance.

43. The mathematically adept reader may remark that since the interval s^2 can take on a negative value, the concept of "space-time" ascribes physical significance to an inherently symbolic "imaginary number." However, the square-root function in symbolic mathematics can be intuitively rendered as the inverse of a compound (duplicate) ratio, such that the problem of the square-root of a negative number is reduced to the simple problem of negative numbers themselves. As mentioned above, negative numbers are intuitively redeemable in terms of a departure from a neutral reference point. In the present case of space-time, the neutral reference point is the null interval of the world-line of a ray of light, from which depart oppositely-signed "time-like" and "space-like" intervals. For this reason, the negative space-time interval does not strike me as problematic.

44. "Homogeneous" here means that time is regarded, and can be treated mathematically, as one more dimension of an overarching "space-time," rather than as an independent dimension over against the three dimensions of space.

45. Husserl, *The Crisis*, pp. 48-9.

46. Hermann Minkowski, "Space and Time," in Hendrik Antoon Lorentz, Albert Einstein, Hermann Minkowski, and Hermann Weyl, *The Principle of Relativity: A Collection of Original Memoirs on the Special and General Theory of Relativity,* trans. W. Perrett and G.B. Jeffery (New York: Dover, 1952), p. 83. "[G]iven by phenomena" here refers to the principle of relativity (no absolute frame of reference is given by phenomena) and the law of the invariability of the velocity of light (which invariability is an empirical law given by phenomena via experiment).

47. David Bohm, *The Special Theory of Relativity* (London: Routledge, 1966), p. 148.

48. Klein, "Modern Rationalism," pp. 63-4.

49. Paul Davies, "That Mysterious Flow," *Scientific American* 27:4 (2002), p. 88.

50. See Augustine, *Confessions* XI. 11.

51. Here, my general conclusion is in line with Hopkins'; see his "Crisis, History, and Husserl's Phenomenological Project."

52. Minkowski, "Space and Time," p. 91.

53. Blumenberg, *The Genesis of the Copernican World,* p. 404.

54. This genetic structure is dialectical. In the philosophical section of his 1905 paper on special relativity, "On the Electrodynamics of Moving Bodies," as well as in other writings, Einstein notes that distant simultaneity is never directly intuited. When we attempt to cash it in indirectly in intuition, it reveals itself as intrinsically connected to an absolute velocity (light), which in turn relativizes simultaneity. Thus does "absolute" time cancel itself dialectically, generating its antithesis "objective-relative" time. However, the co-equal frames of reference that define objective-relative time are themselves arbitrary "accounting devices," and thus demand by their own inner logic an absolute "space-time" constitutive of the truly real world. Viewed dialectically in this manner, absolute space-time is, as Minkowski avers, the only thing truly "given by phenomena." Time as given in the life-world is thus "sublated" (Hegel's *aufgehoben)* in absolute space-time. This highly dialectical genetic structure is clear, for instance, in the opening philosophical section of Einstein's 1905 paper but is ignored as a rule in textbook presentations that begin with the principle of light velocity as an empirical axiom. It is often ignored in philosophical accounts as well. There is a striking parallel between Einstein's treatment of time and Kant's second analogy in the *Critique of Pure Reason*. Kant argues that invariable temporal sequences of event types (cause and effect) constitute an *a priori* condition for the possibility of our distinguishing an objective time sequence from the subjective flow of time. Similarly, Einstein argues that a universal velocity is an *a priori* condition for our distinguishing objective simultaneity from subjectively perceived simultaneity. It is an irony, then, that relativity has been held up as proof against Kantian *a priorism*, for example, by Hans Reichenbach in *The Theory of Relativity and* A Priori *Knowledge,* trans. Maria Reichenbach (Berkeley: University of California Press, 1965). See Thomas Ryckman's

comments on this irony in "Recovering First Philosophy in Philosophy of Physics," *Philosophy Today* 49:5 (2005), pp. 13-22.

55. Heinrich Hertz, quoted in David Park, *The How and the Why* (Princeton: Princeton University Press, 1988), p. 356.